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|  | Lecture Notes | 13th June 2023 |

* Power Series
* 2 Questions for Midterm – 2
* 1 Question for Final Exam

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**Theory**:

A numerical series is of the form:

Example:

* This is a geometric series
* Sum of an infinite geometric series:

* For the example above:
* If you get an answer – The series is said to be convergent.
* If you don’t get an answer – The series is said to be divergent.
* The example above is convergent.
* This example is divergent.

**Power Series:**

A power series (in one variable) is an infinite series of the form:

* A power series is an infinite polynomial.
* The most important example of Power Series is called **Taylor Series**.

Idea:

* The easiest function in mathematics/engineering is a polynomial.
* With Taylor Series we can approximation a function to a polynomial.

Remark:

* Any nice function has a Taylor Series approximation.
* Nice: All derivatives at exist.
* Such a function is called a Smooth Function.

Taylor Series of at center :

* Taylor Series is a special case of Power Series.

**Find Taylor Series of**:

Example 1:

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Application of Power Series:

* Calculator/Computer.

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Example 2: Find

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By Taylor Series,

This is how a Calculator/Computer computes

Application of Power Series:

* Solving ODEs.

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Theorem:

If and have power series approximations then the solution of ODE, y, has a power series approximation.

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Example 3: Solve using power series method

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Here, and are polynomials. Therefore, they have power series approximations

Therefore, solution ‘’ of the ODE also has a power series approximation

Substituting these in the given ODE we get,

Useful Techniques: